LIBERTY PAPER SET

STD. 10 : Mathematics (Standard) [N-012(E)]

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 5**

Section-A

1. (C) 9 **2.** (D) = **3.** (C) $\sqrt{a^2 + b^2}$ **4.** (D) 28 **5.** (B) 5 **6.** (D) 4 **7.** 360 **8.** $-\frac{1}{2}$ **9.** Unique solution **10.** 0 **11.** Circle **12.** $\frac{\pi r^2 \alpha}{360}$ **13.** True **14.** False **15.** True **16.** False **17.** 1 **18.** 2 **19.** Sridharacharya **20.** 0.37 **21.** (b) $\frac{1}{sec\,\theta}$ **22.** (a) $\frac{1}{cosec\,\theta}$ **23.** (b) $2\pi rh$ **24.** (a) $2\pi r(r + h)$

Section-B

25. $85 = 17 \times 5$

$136 = 8 \times 17 = 2^3 \times 17$

HCF (85, 136) = 17

LCM (85, 136) $= 2^3 \times 5 \times 17$

$= 8 \times 85$

$= 680$

26. Let $2x + 3y = 13$ ....(1)

and $4x + 5y = 23$ ....(2)

From (1)

$2x + 3y = 13$

$\therefore\ 2x = 13 - 3y$

$\therefore\ x = \frac{13 - 3y}{2}$ ....(3)

Put in equation (2)

$\therefore\ 4\left(\frac{13 - 3y}{2}\right) + 5y = 23$

$\therefore\ 2(13 - 3y) + 5y = 23$

$\therefore\ 26 - 6y + 5y = 23$

$\therefore\ -y = 23 - 26$

$\therefore\ -y = -3$

$\therefore\ y = 3$

From (3)

$x = \frac{13 - 3(3)}{2}$

$= \frac{13 - 9}{2}$

$= \frac{4}{2}$

$= x = 2$

$\therefore\ x = 2, y = 3$

27. Suppose, the present age of Rohan = x year

The present age of mother = $(x + 26)$ years

After 3 years,

Rohan's age = $(x + 3)$ and his mother's age will

$(x + 26 + 3) = (x + 29)$ years

According to the condition,

$\therefore (x + 3)(x + 29) = 360$

$\therefore x^2 + 29x + 3x + 87 - 360 = 0$

$\therefore x^2 + 32x - 273 = 0$

$x^2 + 39x - 7x - 273 = 0$

$x(x + 39) - 7(x + 39) = 0$

$(x + 39)(x - 7) = 0$

$x + 39 = 0$ OR $x - 7 = 0$

$x = -39$ OR $x = 7$

but x is Rohan's age so negative is not possible

$x \neq -39$

$x = 7$ years

Rohan's Present age is 7 years

and his mother's present age = $x + 26 = 7 + 26 = 33$ years.

28. Here $6x^2 - 13x + 6 = 0$

compare with $ax^2 + bx + c = 0$

$a = 6, b = -13, c = 6$

Discriminant $= b^2 - 4ac$

$= (-13)^2 - 4(6)(6)$

$= 169 - 144$

$= 25$

> 0

$\therefore$ The given quadratic equation has two distinct, real and rational roots.

29. The number of cotton plants in the 1st, 2nd, 3rd, ..., rows are : 23, 21, 19, ..., 5

In the form of AP :

$a = 23, d = 21 - 23 = -2, a_n = 5$

Now, $a_n = a + (n - 1)d$

$\therefore 5 = 23 + (n - 1)(-2)$

$\therefore 5 - 23 = (n - 1)(-2)$

$\therefore \frac{-18}{-2} = n - 1$

$\therefore n - 1 = 9$

$\therefore n = 10$

So, there are 10 rows in the agricultural field.

30. $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1+3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}}$$

$$= \frac{\frac{67}{12}}{1}$$

$$= \frac{67}{12}$$

31. LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + 2 \sin A \operatorname{cosec} A + \operatorname{cosec}^2 A + \cos^2 A + 2 \cos A \sec A + \sec^2 A$

$= \sin^2 A + \cos^2 A + 2 \sin A \operatorname{cosec} A + 2 \cos A \sec A + \operatorname{cosec}^2 A + \sec^2 A$

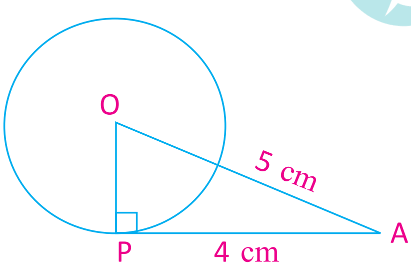
$= 1 + 2(1) + 2(1) + 1 + \cot^2 A + 1 + \tan^2 A$

$= 1 + 2 + 2 + 1 + \cot^2 A + 1 + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A$

= RHS

32.



PA is the tangent to the O-centred circle and P is the tangent PA = 4 cm, OA = 5 cm

In Δ OPA; ∠P = 90° (Theorem : 10.1)

$\therefore OP^2 + PA^2 = OA^2$

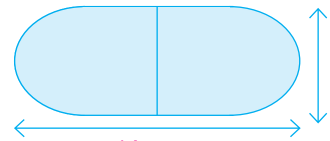
$\therefore OP^2 = OA^2 - PA^2 = (5)^2 - (4)^2 = 25 - 16$

$\therefore OP^2 = 9$

$\therefore OP = 3$ cm

Hence, the radius of the circle is 3 cm.

33. Cylinder Hemisphere



$d = 5.5$ mm $\quad r = \frac{5}{2} = 2.5$ mm

$\therefore r = \frac{5}{2} = 2.5$ mm

Height of cylinder h = Length of capsule – 2 × Radius of hemisphere

$\therefore h = 14 - (2 \times 2.5)$

$\therefore h = 14 - 5$

$\therefore h = 9$ mm

Surface area of capsule

= CSA of cylinder + 2 × CSA of hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r\,(h + 2r)$

$= 2 \times \frac{22}{7} \times 2.5 \times [9 + 2(2.5)]$

$= 2 \times \frac{22}{7} \times 2.5 \times (9 + 5)$

$= 5 \times \frac{22}{7} \times 14$

$= 5 \times 22 \times 2$

$= 220$ mm^2

34. Here, the maximum number of students i.e. 7 have got marks in the interval 40 – 55, the modal class is 40 – 55.

$\therefore$ l = the lower limit of the modal calss = 40

h = class size = 15

f_1 = the frequency of modal class = 7

f_0 = the frequency of modal class preceding the modal class = 3

f_2 = the frequency of the class succeeding the modal class = 6

$$\text{Mode } Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\therefore Z = 40 + \left(\frac{7 - 3}{2(7) - 3 - 6}\right) \times 15$$

$$\therefore Z = 40 + \frac{4 \times 15}{5}$$

$\therefore Z = 40 + 12$

$\therefore Z = 52$

35. We have, $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 50 + \frac{-36}{35} \times 10$

$= 50 - \frac{36 \times 10}{7 \times 5}$

$= 50 - \frac{36 \times 2}{7}$

$= 50 - \frac{72}{7}$

$= 50 - 10.28$

$\bar{x} = 39.72$

36. Here total number of cards = 52

(i) Suppose A be the event "the card is a black card"

$\therefore$ The number of black cards = 13

$\therefore$ The number of outcomes favourable to A = 13

$\therefore$ P (A) = $\frac{13}{52} = \frac{1}{4}$

(ii) Suppose B be the event "the card is a red face card"

$\therefore$ The number of red face cards = 6

$\therefore$ The number of outcomes favourable to B = 6

$\therefore$ P (B) = $\frac{6}{52} = \frac{3}{26}$

37. There are only lemon-flavoured desserts in one bag. If we take the number of lemon flavoured sweets = n, then the total number of results of the experiment is = n. But the number of orange flavoured desserts = 0.

(i) Suppose, the selected dersert has an orange flavour, let's call that event A,

$$P(E) = \frac{\text{Number of dessert to taste orange}}{\text{The total number of results from the experiment}}$$

$\therefore$ P(A) = $\frac{0}{n}$

$\therefore$ P(A) = 0

(ii) Suppose, the chosen dessert tastes like lemon, let's call that event B,

$$P(B) = \frac{\text{Number of desserts to taste lemon}}{\text{The total number of results from the experiment}}$$

$\therefore$ P(B) = $\frac{n}{n}$

$\therefore$ P(B) = 1

Section-C

38. Here P (x) = $6x^2 - 13x + 6$

compare with P(x) = $ax^2 + bx + c$

$a = 6,\ b = -13,\ c = 6$

$\alpha + \beta = \frac{-b}{a} = \frac{(-13)}{6} = \frac{13}{6}$

$\alpha \cdot \beta = \frac{c}{a} = \frac{6}{6} = 1$

(i) $\alpha^2 + \beta^2$

$$\alpha^2 + \beta^2 = \alpha^2 + 2\alpha\beta + \beta^2 - 2\alpha\beta$$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{13}{6}\right)^2 - 2\,(1)$$

$$= \frac{169}{36} - 2$$

$$= \frac{169 - 72}{36}$$

$$\therefore \alpha^2 + \beta^2 = \frac{97}{36}$$

(ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$= \frac{97/36}{1}$$

$$\therefore \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{97}{36}$$

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$

$$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$$

$$= \frac{\alpha+\beta}{\alpha\beta}$$

$$= \frac{13/6}{1}$$

$$\therefore \frac{1}{\alpha}+\frac{1}{\beta} = \frac{13}{6}$$

39. Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α & β.

$\therefore \alpha+\beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{3} = \frac{c}{a}$

$\therefore a = 3,\ b = -3\sqrt{2}$ and $c = 1$

So, one quadratic polynomial which fits the given conditions is $3x^2 - 3\sqrt{2}x + 1$. You can check that any other quadratic polynomial that fits these conditions will be of the form $k(3x^2 - 3\sqrt{2}x + 1)$, where k is real.

40. Here AP will be 7, 14, 21,... 140

Becaouse last term is 20[th] term which is multiple of 7. $\therefore$ 7 × 20 = 140.

a = 7, n = 20, l = 140

$S_n = \frac{n}{2}[a + l]$

$\therefore\ S20 = \frac{20}{2}[7 + 140]$

$\therefore\ S20 = 1470$

So, sum of first 20 multiples of 7 is 1470.

41. $a = 5,\ a_n = l = 45,\ S_n = 400,\ n =$ ____, $d =$ ____

$S_n = \frac{n}{2}(a + l)$

$\therefore 400 = \frac{n}{2}(5 + 45)$

$\therefore 800 = n \times 50$

$\therefore n = \frac{800}{50}$

$\therefore n = 16$

Now, $a_n = a + (n-1)d$

$\therefore 45 = 5 + (16-1)d$

$\therefore 45 - 5 = 15d$

$\therefore 40 = 15d$

$\therefore d = \frac{40}{15}$

$\therefore d = \frac{8}{3}$

42. Suppose, the ratio in which line segment joining A (–3, 10) and B (6, – 8) is divided by point P (–1, 6) is $m_1 : m_2$.

Co-ordinates of point P = $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$\therefore$ (–1, 6) = $\left(\frac{m_1(6) + m_2(-3)}{m_1 + m_2}, \frac{m_1(-8) + m_2(10)}{m_1 + m_2}\right)$

$\therefore$ (–1, 6) = $\left(\frac{6m_1 - 3m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$

$\therefore -1 = \frac{6m_1 - 3m_2}{m_1 + m_2}$

$\therefore -m_1 - m_2 = 6m_1 - 3m_2$

$\therefore -m_1 - 6m_1 = -3m_2 + m_2$

$\therefore -7m_1 = -2m_2$

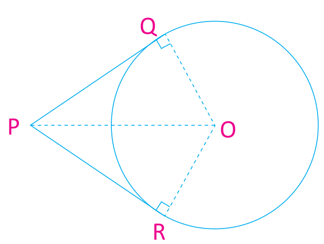
$\therefore \frac{m_1}{m_2} = \frac{2}{7}$

Hence, the point P will divide AB into a 2 : 7 ratio.

43. Given : A circle with centre O, a point P lying outside the circle with two tangents PQ, QR on the circle from P.

To prove : PQ = PR

Figure :



Proof : Join OP, OQ and OR. Then ∠OQP and ∠ORP are right angles because these are angles between the radii and tangents and according to theorem 10.1 they are right angles.

Now, in right triangles OQP and ORP,

OQ = OR (Radii of the same circle)

OP = OP (Common)

∠OQP = ∠ORP (Right angle)

Therefore, Δ OQP ≅ Δ ORP (RHS)

This gives, PQ = PR (CPCT)

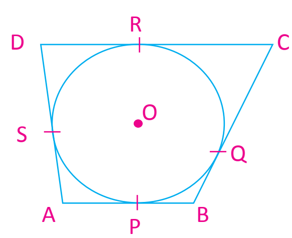
44. Let the sides AB, BC, CD and DA of the quadrilateral ABCD touch the O centric circle at points P, Q, R and S respectively.

$\therefore$ AP = AS ...(1)

BP = BQ ...(2)

CR = CQ ...(3)

DR = DS ...(4)



Add equation (1), (2), (3) and (4)

AP + BP + CR + DR = AS + BQ + CQ + DS

$\therefore$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\therefore$ AB + CD = AD + BC

45. $\theta = 115°$

r = length of blade = 25 cm

Area of minor sector $= \dfrac{\pi r^2 \theta}{360}$

$= \dfrac{22 \times 25 \times 25 \times 115}{7 \times 360}$

$= \dfrac{1581250}{2520}$

$= \dfrac{158125}{252}$ cm^2

$\therefore$ Area swept by 2 blades

= 2 × Area of minor sector

$= 2 \times \dfrac{158125}{252}$

$= \dfrac{158125}{126}$ cm^2

46. Total cards in deck = 52

A : card is an Ace = 4

B : card is not an Ace = 48

C : card is Red colour Ace = 2

We have formula,

$$P(E) = \frac{\text{No. of possible outcomes for given event}}{\text{Total outcomes}}$$

So,

$$P(A) = \frac{4}{52} = \frac{4}{4 \times 13} = \frac{1}{13}$$

$$P(B) = \frac{48}{52} = \frac{12 \times 4}{13 \times 4} = \frac{12}{13}$$

$$P(C) = \frac{2}{52} = \frac{2 \times 1}{13 \times 4} = \frac{1}{26}$$

Section-D

47. Let, correct questions = x

Wrong question = y

Condition 1 : $3x - y = 40$ ...(1)

Condition 2 : $4x - 2y = 50$ ...(2)

From eqn (1) $y = 3x - 40$ ...(3)

Put value of (3) in (2),

$4x - 2(3x - 40) = 50$

$\therefore$ $4x - 6x + 80 = 50$

$\therefore$ $-2x = 50 - 80$

$\therefore$ $-2x = -30$

$\therefore$ $-x = \dfrac{-30}{2}$

$\therefore$ $x = 15$

Put $x = 15$ in (3),

y = 3(15) – 40

= 45 – 40

y = 5

Total question = x + y

= 15 + 5

= 20

$\therefore$ Total question in exam = 20

48. Suppose, the large number is x and the smaller number is y.

$\therefore\ x - y = 26$ ...(1)

$x = 3y$ ...(2)

Put value of equation (2) in equation (1),

$x - y = 26$

$\therefore\ 3y - y = 26$

$\therefore\ 2y = 26$

$\therefore\ y = 13$

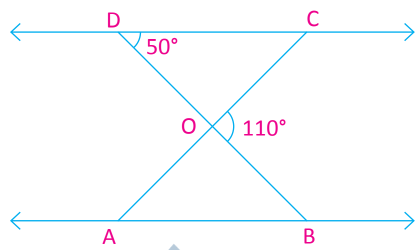
Put $y = 13$ in equation (2),

$x = 3y$

$\therefore\ x = 3 \times 13$

$\therefore\ x = 39$

Therefore, the numbers are 39 and 13.



49. (i) $\angle DOC + \angle BOC = 180°$

$\therefore\ \angle DOC + 110° = 180°$

$\therefore\ \angle DOC = 180° - 110°$

$\therefore\ \angle DOC = 70°$

In Δ ODC, $\angle CDO + \angle DCO + \angle DOC = 180°$

$\therefore\ 50° + \angle DCO + 70° = 180°$

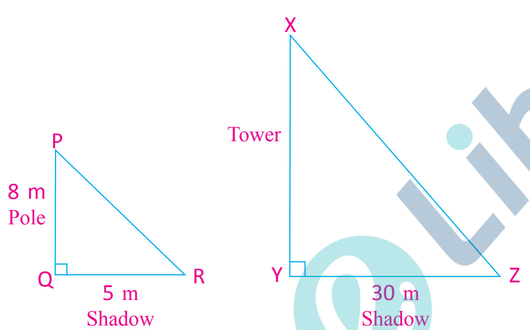
$\therefore\ 120° + \angle DCO = 180°$

$\therefore\ \angle DCO = 180° - 120°$

$\therefore\ \angle DCO = 60°$

Hence, $\angle DCO = 70°$ and $\angle DCO = 60°$

(ii)



Here, in Δ PQR, PQ is a pole and QR is a shadow of the pole.

$\therefore$ PQ = 8 m and QR = 5 m

In Δ XYZ, XY is a tower and YZ is a shadow of the tower.

$\therefore$ YZ = 30 m

The length of both shadows are measured at a same time, so $\angle R$ and $\angle Z$ are the angles of elevation of the sun.

$\therefore\ \angle R = \angle Z$

In Δ PQR and Δ XYZ,

$\angle R = \angle Z$ and $\angle Q = \angle Y$ (Right angles)

$\therefore\ \Delta PQR \sim \Delta XYZ$ (AA criterion)

$$\therefore\ \frac{PQ}{XY} = \frac{QR}{YZ}$$

$$\therefore\ \frac{8}{XY} = \frac{5}{30}$$

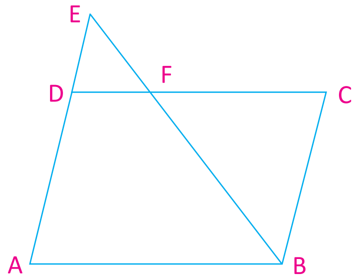
$$\therefore\ XY = \frac{8 \times 30}{5}$$

$\therefore$ XY = 48 m

Therefore, the height of the tower = 48 m.

50. Given : E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F.

To Prove : Δ ABE ~ Δ CFB



Proof : In parallelogram ABCD,

∠BAD = ∠DCB (opposite angles)

∴ ∠BAE = ∠FCB ...(1)

Point E is the point on the extended side AD of the parallelogram ABCD.

∴ AE || BC

∴ ∠AEB = ∠EBC (Alternate interior angles)

∴ ∠AEB = ∠FBC ...(2)

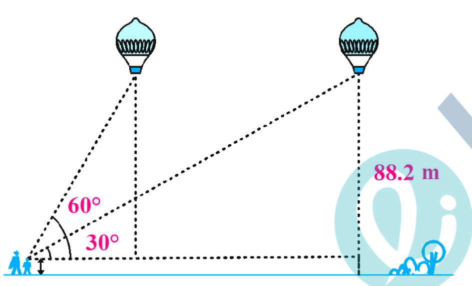
In Δ ABE and Δ CFB,

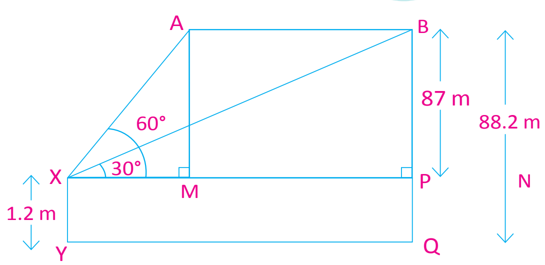
∠BAE = ∠FCB (∵ As per equation (1))

∠AEB = ∠FBC (∵ As per equation (2))

∴ Δ ABE ~ Δ CFB (AA criterion)

51.





Here, the location of the A and B balloons, XY = height, YQ ground and XP is the horizontal line from the girl's eyes.

Take, AM ⊥ XP, M is the point on XP.

Therefore, in Δ AMX; ∠AMX = 90° and ∠AXM = 60°,

In Δ BPX, ∠BPX = 90°, ∠BXP = 30°,

PQ = XY = 1.2 m, BQ = 88.2 m

∴ AM = BP = BQ – PQ = 88.2 – 1.2 = 87 m

In Δ AMX; ∠AMX = 90°

$\therefore\ \tan 60° = \frac{AM}{XM}$

$\therefore\ \sqrt{3} = \frac{87}{XM}$

$\therefore\ XM = \frac{87}{\sqrt{3}}$

$\therefore\ XM = 29\sqrt{3}$

In Δ BPX; $\angle BPX = 90°$

$\therefore\ \tan 30° = \frac{BP}{XP}$

$\therefore\ \frac{1}{\sqrt{3}} = \frac{87}{XP}$

$\therefore\ XP = 87\sqrt{3}$

Now, MP $= XP - XM$

$= 87\sqrt{3} - 29\sqrt{3}$

$= 58\sqrt{3}$ m

$\therefore\ AB = 58\sqrt{3}$ m

Hence, the distance travelled by balloon is $58\sqrt{3}$ m.

52. Hemisphere Cone

$r = 3.5$ cm $\quad r = 3.5$ cm

$h = 12$ cm

$l = 12.5$ cm

Total height of the toy = 15.5 cm

Height of cone + Radius of hemisphere = 15.5

$\therefore\ h + 3.5 = 15.5$

$\therefore\ h = 12$ cm

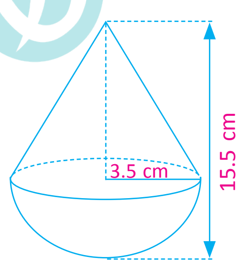
Now, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(3.5)^2 + (12)^2}$

$= \sqrt{12.25 + 144}$

$l = \sqrt{156.25}$

$\therefore\ l = 12.5$ cm



Total surface area of toy

= CSA of hemisphere + CSA of cone

$= 2\pi r^2 + \pi rl$

$= \pi r(2r + l)$

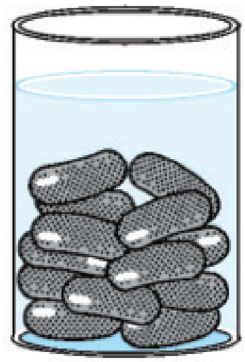
$= \frac{22}{7} \times 3.5 \times [2(3.5) + 12.5]$

$= 22 \times 0.5 \times (7 + 12.5)$

$= 11 \times 19.5$

$= 214.5\ cm^2$

53.

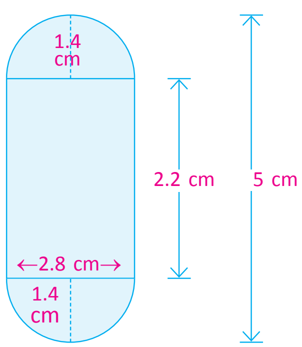


Cylinder	Hemisphere
$d = 2.8$ cm	$d = 2.8$ cm
$\therefore\ r = 1.4$ cm	$\therefore\ r = 1.4$ cm
Height of cylinder	h = Totat length $- 2r$

$\therefore\ h = 5 - 2(1.4)$

$\therefore\ h = 5 - 2.8$

$\therefore\ h = 2.2$ cm



$\therefore$ Volume of 45 gulabjamun = 45 × Volume of one gulabjamun

= 45 × (Volume of cylinder + 2 × Volume hemisphere)

$= 45 \times (\pi r^2 h + 2 \times \frac{2}{3}\pi r^3)$

$= 45 \times (\pi r^2 h + \frac{4}{3}\pi r^3)$

$= 45 \times \pi r^2 \times (h + \frac{4}{3} r)$

$= 45 \times \frac{22}{7} \times (1.4)^2 \times \left(2.2 + \frac{4 \times 1.4}{3}\right)$

$= 45 \times \frac{22}{7} \times 1.96 \times \left(\frac{6.6 + 5.6}{3}\right)$

$= 45 \times 22 \times 0.28 \times \frac{12.2}{3}$

$= 15 \times 22 \times 0.28 \times 12.2$

$= 1127.28\ \text{cm}^3$

$\therefore$ Volume of sugar syrup = 30% of volume

$= 1127.28 \times \frac{30}{100}$

$= 338.184\ \text{cm}^3$

$= 338\ \text{cm}^3$ (Approx)

54.

Class intervals	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	f_1	$7 + f_1$
300 – 400	12	$19 + f_1$
400 – 500	17	$36 + f_1$
500 – 600	20	$56 + f_1$
600 – 700	f_2	$56 + f_1 + f_2$
700 – 800	9	$65 + f_1 + f_2$
800 – 900	7	$72 + f_1 + f_2$
900 – 1000	4	$76 + f_1 + f_2$

It is given that $n = 100$

$$\frac{n}{2} = \frac{100}{2} = 50$$

$\therefore\ 76 + f_1 + f_2 = 100$

$\therefore\ f_1 + f_2 = 24$

The median is 525, which lies in the class 500 – 600.

$l = 500$

$cf = 36 + f_1$

$f = 20$

$h = 100$

$$\text{Median M} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore\ 525 = 500 + \left(\frac{50 - 36 - f_1}{20}\right) \times 100$$

$\therefore\ 525 - 500 = (14 - f_1)5$

$$\therefore\ \frac{25}{5} = 14 - f_1$$

$\therefore\ 5 = 14 - f_1$

$\therefore\ f_1 = 14 - 5$

$\therefore\ f_1 = 9$

Now, $f_1 + f_2 = 24$

$\therefore\ 9 + f_2 = 24$

$\therefore\ f_2 = 15$